

## ABSTRAK

Spektrum graf adalah susunan nilai eigen dari matriks ketetanggaan beserta multiplisitasnya. Spektrum yang dihasilkan dari matriks *Laplace* dinamakan spektrum *Laplace*. Matriks *Laplace* dari suatu graf merupakan selisih matriks diagonal dan matriks ketetanggaan. Skripsi ini membahas penentuan bentuk umum polinomial karakteristik dan bentuk umum spektrum *Laplace* pada graf-graf reguler diantaranya graf bipartit lengkap ( $K_{n,n}$ ), graf mahkota ( $S_n^0$ ), dan graf benteng ( $B_{nn}$ ). Hasil penelitian diperoleh bentuk umum polinomial karakteristik matriks *Laplace* dari graf bipartit lengkap ( $K_{n,n}$ ) dengan  $n > 1$  yaitu  $p(\lambda) = \lambda(\lambda - 2n)(\lambda - n)^{2(n-1)}$ , graf mahkota ( $S_n^0$ ) dengan  $n \geq 3$  yaitu  $p(\lambda) = \lambda(\lambda - (2n - 2))(\lambda - (n - 2))^{n-1}(\lambda - n)^{n-1}$ , dan graf benteng ( $B_{nn}$ ) dengan  $n \geq 2$  yaitu  $p(\lambda) = \lambda(\lambda - 2n)^{(n-1)^2}(\lambda - n)^{2(n-1)}$ . Selanjutnya, dari polinomial karakteristik graf tersebut dapat diperoleh bentuk umum spektrum *Laplace* untuk graf bipartit lengkap ( $K_{n,n}$ ) dengan  $n > 1$ , graf mahkota ( $S_n^0$ ) dengan  $n \geq 3$ , dan graf benteng ( $B_{nn}$ ) dengan  $n \geq 2$ , yaitu

$$\text{spec}_L(K_{n,n}) = \begin{bmatrix} 0 & n & 2n \\ 1 & 2(n-1) & 1 \end{bmatrix}, \text{spec}_L(S_n^0) = \begin{bmatrix} 0 & n-2 & n & 2n-2 \\ 1 & n-1 & n-1 & 1 \end{bmatrix},$$

dan  $\text{spec}_L(B_{nn}) = \begin{bmatrix} 0 & n & 2n \\ 1 & 2(n-1) & (n-1)^2 \end{bmatrix}$ .

**Kata kunci:** graf bipartit lengkap, graf mahkota, graf benteng, polinomial karakteristik, spektrum *Laplace*.

## ABSTRACT

The spectrum of a graph is a set of eigenvalues of the adjacency matrix along with their multiplicities. The spectrum of a Laplace matrix is called the Laplace spectrum. The Laplace matrix of a graph is the difference between the diagonal matrix and the adjacency matrix. This research studied the definition of the general form of the characteristic polynomial and the general form of the Laplace spectrum on regular graphs, which are the complete bipartite graph  $(K_{n,n})$ , the crown graph  $(S_n^0)$ , and the rook's graph  $(B_{nn})$ . This study found that the general form of the Laplace matrix's characteristic polynomial for the complete bipartite graph  $(K_{n,n})$  with  $n > 1$  is  $p(\lambda) = \lambda(\lambda - 2n)(\lambda - n)^{2(n-1)}$ , the crown graph  $(S_n^0)$  with  $n \geq 3$  is  $p(\lambda) = \lambda(\lambda - (2n - 2))(\lambda - (n - 2))^{n-1}(\lambda - n)^{n-1}$ , and the rook's graph  $(B_{nn})$  with  $n \geq 2$  is  $p(\lambda) = \lambda(\lambda - 2n)^{(n-1)^2}(\lambda - n)^{2(n-1)}$ . Furthermore, based on the graphs' characteristic polynomials, the general form of the Laplace spectrum for the complete bipartite graph  $(K_{n,n})$  with  $n > 1$ , the crown graph  $(S_n^0)$  with  $n \geq 3$ , and the rook's graph  $(B_{nn})$  with  $n \geq 2$ , are

$$\text{spec}_L(K_{n,n}) = \begin{bmatrix} 0 & n & 2n \\ 1 & 2(n-1) & 1 \end{bmatrix}, \text{spec}_L(S_n^0) = \begin{bmatrix} 0 & n-2 & n & 2n-2 \\ 1 & n-1 & n-1 & 1 \end{bmatrix},$$

and  $\text{spec}_L(B_{nn}) = \begin{bmatrix} 0 & n & 2n \\ 1 & 2(n-1) & (n-1)^2 \end{bmatrix}$ .

**Keywords:** complete bipartite graph, crown graph, rook's graph, characteristic polynomial, Laplace spectrum.