ABSTRAK

Susunan nilai eigen dari matriks ketetanggaan beserta multiplisitasnya disebut spektrum graf. Spektrum graf yang dihasilkan dari matriks *distance Laplacian* disebut sebagai spektrum *distance Laplacian*, sedangkan spektrum yang dihasilkan dari matriks *distance Signless Laplacian* disebut spektrum *distance Signless Laplacian*. Matriks *distance Laplacian* dari suatu graf merupakan selisih dari matriks transmisi dan matriks *distance*, sedangkan matriks *distance Signless Laplacian* dari suatu graf merupakan penjumlahan dari matriks transmisi dan matriks *distance*. Penelitian ini bertujuan untuk menentukan bentuk umum polinomial karakteristik dan bentuk umum spektrum *distance Laplacian* serta *distance Signless Laplacian* pada graf bipartit lengkap $\left(K_{n,n}\right)$ dan graf tripartit lengkap $\left(K_{n,n,n}\right)$ dengan $n \geq 2$. Polinomial karkateristik yang dihasilkan diperoleh dari perkalian elemen-elemen diagonal utama pada matriks segitiga atas yang merupakan hasil reduksi persamaan karakteristik dengan Eliminasi Gaussian. Hasil penelitian ini diperoleh bentuk umum polinomial karakteristik matriks *distance Laplacian* pada graf bipartit lengkap $\left(K_{n,n}\right)$ dengan $n \geq 2$ adalah $p(\mu) = \mu(\mu - 2n)(\mu - 3n)^{2n-2}$, sedangkan untuk graf tripartit lengkap $\left(K_{n,n,n}\right)$ dengan $n \geq 2$ adalah $p(\mu) = \mu(\mu - 3n)^2(\mu - 4n)^{3n-3}$. Bentuk umum polinomial karakteristik matriks *distance Signless Laplacian* pada graf bipartit lengkap $\left(K_{n,n}\right)$ dengan $n \geq 2$ adalah $p(\delta) = \left(\delta - (3n - 4)\right)^{2n-2}\left(\delta - (4n - 4)\right)\left(\delta - (6n - 4)\right)$, sedangkan untuk graf tripartit lengkap $\left(K_{n,n,n}\right)$ dengan $n \geq 2$ adalah $p(\delta) = \left(\delta - (4n - 4)\right)^{3n-3}\left(\delta - (5n - 4)\right)^2\left(\delta - (8n - 4)\right)$. Selanjutnya, dari polinomial karakteristik graf tersebut dapat diperoleh bentuk umum spektrum *distance Laplacian* dan spektrum *distance Signless Laplacian* pada graf bipartit lengkap $\left(K_{n,n}\right)$ dan graf tripartit lengkap $\left(K_{n,n,n}\right)$, dengan $n \geq 2$, yaitu $Spec_{L_D}K_{n,n} = \begin{bmatrix} 0 & 2n & 3n \\ 1 & 1 & 2n-2 \end{bmatrix}$, $Spec_{L_D}K_{n,n,n} = \begin{bmatrix} 0 & 3n & 4n \\ 1 & 2 & 3n-3 \end{bmatrix}$, $Spec_{Q_D}K_{n,n} = \begin{bmatrix} 3n-4 & 4n-4 & 6n-4 \\ 2n-2 & 1 & 1 \end{bmatrix}$, dan $Spec_{Q_D}K_{n,n,n} = \begin{bmatrix} 4n-4 & 5n-4 & 8n-4 \\ 3n-3 & 2 & 1 \end{bmatrix}$.

Kata kunci : graf bipartit lengkap, graf tripartit lengkap, polinomial karakteristik, spektrum *distance Laplacian*, spektrum *distance Signless Laplacian*.

ABSTRACT

A set of eigenvalues of the adjacency matrix along with their multiplicities si called the spectrum of a graph. The spectrum of a distance Laplacian matrix is called the distance Laplacian spectrum, while the spectrum of a distance Signless Laplacian matrix is called the distance Signless Laplacian spectrum. The distance Laplacian matrix of a graph is the difference between the transmission matrix and the distance matrix, while the distance Signless Laplacian matrix of a graph is the sum between the transmission matrix and the distance matrix. The purpose of this research study is to determine the general form of the characteristic polynomial and the general form of the distance Laplacian spectrum and distance Signless Laplacian spectrum of the complete bipartite graphs $(K_{n,n})$ *and the complete tripartite graphs* $(K_{n,n,n})$ *with* $n \geq 2$*. The resulting characteristic polynomial is obtained from the multiplication of the main diagonal elements in the upper triangular matrix which is the result of the reduction of the characteristic equation by Gaussian elimination. The results of this study found that the general form of the distance Laplacian matrix characteristic polynomial for the complete bipartite graphs* $(K_{n,n})$ *with* $n \geq 2$ *is* $p(\mu) = \mu(\mu - 2n)(\mu - 3n)^{2n-2}$*, while for the complete tripartite graphs* $(K_{n,n,n})$ *with* $n \geq 2$ *is* $p(\mu) = \mu(\mu - 3n)^2(\mu - 4n)^{3n-3}$*. The general form of the distance Signless Laplacian matrix characteristic polynomial for the complete bipartite graphs* $(K_{n,n})$ *with* $n \geq 2$ *is* $p(\delta) = (\delta - (3n-4))^{2n-2}(\delta - (4n-4))(\delta - (6n-4))$*, and* $p(\delta) = (\delta - (4n-4))^{3n-3}(\delta - (5n-4))^2(\delta - (8n-4))$ *for the complete tripartite graphs* $(K_{n,n,n})$ *with* $n \geq 2$*. Furthermore, based on the graph characteristic polynomial, the general form of the distance Laplacian spectrum and the distance Signless Laplacian spectrum for the complete bipartite graph* $(K_{n,n})$ *and the complete tripartite graph* $(K_{n,n,n})$*, with* $n \geq 2$ *are* $Spec_{L_D} K_{n,n} = \begin{bmatrix} 0 & 2n & 3n \\ 1 & 1 & 2n-2 \end{bmatrix}$, $Spec_{L_D} K_{n,n,n} = \begin{bmatrix} 0 & 3n & 4n \\ 1 & 2 & 3n-3 \end{bmatrix}$, $Spec_{Q_D} K_{n,n} = \begin{bmatrix} 3n-4 & 4n-4 & 6n-4 \\ 2n-2 & 1 & 1 \end{bmatrix}$, *and* $Spec_{Q_D} K_{n,n,n} = \begin{bmatrix} 4n-4 & 5n-4 & 8n-4 \\ 3n-3 & 2 & 1 \end{bmatrix}$.

***Keywords**: complete bipartite graph, complete tripartite graph, characteristic polynomial, distance Laplacian spectrum, distance Signless Laplacian spectrum.*